Crédo Roselin Fanou

Title: Prescribed Exponential Decay of the Wave Equation via Internal Delayed Potential

Abstract. In this study, we investigate the stabilization of the wave equation using an internal delayed potential. Building on the recently developed partial pole placement paradigm—originally introduced in the context of functional differential equations—we aim to design a feedback law that achieves globally optimal exponential decay of the closed-loop system solution. For this purpose, we first establish the well-posedness of the problem using the semigroup approach via a shifting operator. Then, we study the asymptotic behavior of the semigroup generated by this operator. The main result of this presentation is a necessary and sufficient condition, expressed in the parameter plane, for the system's spectrum to lie in the left half-plane of the complex plane. This will be illustrated by a diagram and several descriptive plots.

The system under consideration is given by:

$$\begin{cases} u_{tt}(x,t) - u_{xx}(x,t) + u(x,t) = 0, & x \in (0,\ell),\ t > 0, \\ u(x,t) = \beta\, u(x,t-\tau), & \\ u(0,t) = 0 = u(\ell,t), & t > 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in (0,\ell), \\ u(x,t-\tau) = f_0(t-\tau), & x \in (0,\ell),\ t \in (0,\tau). \end{cases}$$

The constant $\tau > 0$ denotes the time-delay and $\ell > 0$, β are real numbers with $\beta \neq 0$. The initial data u_0, u_1, and f_0 are given functions belonging to suitable spaces that will be specified later. The delay is considered here as a control parameter.